

Dualising the Dual Standard Model

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We discuss how the dual standard model and the dualised standard model are complementary theories. That is, how their implications have no overlap, whilst together they explain most features of the standard model. To illustrate how these two theories might be combined we consider the dual standard model in a theta vacuum. Whilst there are issues to be considered, the dual standard model does then appear to become naturally dualised. This supports an origin of a dual formulation of the standard model through the properties of $SU(5)$ solitons in a theta vacuum.

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I. INTRODUCTION

Recently a remarkable correspondence has been discovered between the monopoles from Georgi-Glashow gauge unification and the observed elementary particles. Vachaspati found that the magnetic charges of the five stable $SU(5)$ monopoles have a one-to-one identification with the electric charges of the five multiplets in one standard model generation [1]. Motivated by this he conjectured that the elementary particles may originate as solitons from $SU(5)$ gauge unification.

A concrete way for examining this conjecture has been proposed by Liu and Vachaspati in the form of the *dual standard model* [2]. This relies on the notion, familiar from electromagnetism, that electric particles can also be described by monopoles in the dual gauge potential. In this sense the dual standard model would be the dual description of the standard model, with all of the elementary particles represented instead as monopoles.

By expressing the standard model in this dual formulation it is possible that there may emerge features that are presently hidden within the usual particle description. That is, a dual standard model may uncover a hidden simplicity and regularity of form that could prove crucial to understanding the nature and origin of the standard model. Also possible is that new physics may have to be included to arrive at a simple and consistent form.

This discovery of the $SU(5)$ monopole-particle correspondence strongly hints that a dual standard model should be formulated around the monopoles from gauge unification. In Vachaspati's words [1]: *This correspondence suggests that perhaps unification should be based on a magnetic $SU(5)$ symmetry group with only a bosonic sector and the presently observed fermions are really the monopoles of that theory.* Much work still needs to be done on this proposal, but several encouraging features do occur. For instance the incorporation of spin [3] and a consistent picture of confinement [2,4].

In this paper we are concerned with the construction

of the dual standard model and whether it could be naturally dualised, in the sense of Chan and Tsou [5]. In their *dualised standard model* [6] (which should not be confused with the dual standard model) they interpret many properties of the elementary particles as emerging from duality; for instance three generations arise from just one generation of dyons. Remarkably this gives accurate estimations for both the masses and mixing angles of the elementary particles [7].

A central point of this paper is that both the methodology and the conclusions of the dual and dualised standard models appear to be complementary to each other. That is, there is no overlap in their conclusions, whilst taking the two models together appears to explain most observed features of the elementary particles. For this reason we examine whether these two models could be considered together.

To illustrate how these models might be combined we investigate the dual standard model in a theta vacuum. Whilst there are issues to be considered, it appears that the initial assumptions of the dualised standard model can emerge. In this sense the dual standard model becomes naturally dualised. Also, giving further corroboration, this calculation appears to explain the chirality assignments of the elementary fermions; a feature that cannot be derived in either of the original models.

If the above two models can be combined together in such a simple and natural way then perhaps a very simple theory of particle and gauge unification could ensue. Indeed it seems possible that *all features of a dual standard model could naturally emerge within the properties of $SU(5)$ monopoles in a theta vacuum.* As we have mentioned such a behaviour does seem to be occurring. However more research is necessary to determine whether this can be fully realised.

The composition of this paper is as follows. In sec. (II) we briefly discuss the dual and dualised standard models and how they relate to each other. Then in sec. (III) we discuss how the two models may naturally combine in a theta vacuum. Finally in sec. (IV) we draw our conclusions.

Before starting we note that an alternative viewpoint for realising a dual standard model has been presented

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by Vachaspati and Steer [8].

II. DUALITY AND THE STANDARD MODEL

In this section we quickly remind the reader of some results within the dual standard model and the dualised standard model. This discussion is also intended to clarify the complementary roles these theories presently take.

A. The Dual Standard Model

The construction of a dual standard model is based around a Georgi-Glashow unification [9] of the standard model gauge symmetry within an $SU(5)$ group *

$$SU(5) \rightarrow H_{SM} = SU(3)_C \times SU(2)_I \times U(1)_Y / \mathbb{Z}_6, \quad (1)$$

which breaks via condensation [11] of an adjoint scalar field. This implies a spectrum of stable $SU(5)$ monopoles, having various colours, isospins and hypercharges. Their magnetic charges are specified by the magnetic field

$$\mathbf{B} \sim \frac{1}{2g} \frac{\hat{\mathbf{r}}}{r^2} M, \quad M = m_C T_C + m_I T_I + m_Y T_Y, \quad (2)$$

with a suitable choice of generators, for instance,

$$T_C = \text{diag}(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, 0, 0), \quad T_I = \text{diag}(0, 0, 0, -1, 1), \\ T_Y = \text{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}). \quad (3)$$

When the scalar masses are much smaller than the gauge masses Gardner and Harvey showed there are five, topologically distinct, stable monopoles with magnetic charges forming the pattern [12]:

TABLE I. $SU(5)$ monopole charges.

topology n	$\text{diag } M$	m_C	m_I	m_Y	multiplet
1	(0, 0, 1, -1, 0)	1	$\frac{1}{2}$	$\frac{1}{3}$	$(u, d)_L$
2	(0, 1, 1, -1, -1)	-1	0	$\frac{2}{3}$	\bar{d}_L
3	(1, 1, 1, -2, -1)	0	$-\frac{1}{2}$	1	$(\bar{\nu}, \bar{e})_R$
4	(1, 1, 2, -2, -2)	1	0	$\frac{4}{3}$	u_R
6	(2, 2, 2, -3, -3)	0	0	2	\bar{e}_L

Based upon this a dual standard model could be constructed along the following lines:

(i) First and foremost the magnetic charges in table I are identical to the electric charges in one standard model

*Note that (1) relies on the elementary particles forming representations of $SU(3) \times SU(2) \times U(1)/\mathbb{Z}_6$; as is implied by an observed \mathbb{Z}_6 relation between their colour, isospin and hypercharge assignments [10].

generation [1]. This suggests that one generation of standard model particles have a monopole description as solitons from a dual $\widetilde{SU}(5)$ unification of the magnetic gauge symmetry $\widetilde{H}_{SM} = \widetilde{SU}(3)_C \times \widetilde{SU}(2)_I \times \widetilde{U}(1)_Y / \mathbb{Z}_6$ in the dual standard model.

(ii) To represent standard model fermions these solitons should have an intrinsic one-half angular momentum. This can be naturally achieved through the fermions from bosons effect [13,14]; from which the dyons formed from combining $SU(5)$ monopoles and quanta of a **5** scalar field H have the requisite angular momenta [3].

(iii) Confinement is expressed through breaking dual colour $\widetilde{SU}(3)_C \rightarrow \mathbb{Z}_3$ [2,4], which attaches the appropriate monopoles to topological vortices.

(iv) When normalising the generators (3) to $\text{tr } T^2 = 1$ the gauge-monopole couplings naturally scale within the minimal coupling $g A_\mu^a T^a$. This suggests the dual standard model unifies when $\frac{1}{3} g_C = g_I = \sqrt{\frac{15}{2}} g_Y$ [15]. Curiously such scaled coupling do unify, although the scale of unification is rather low and could prove problematic:

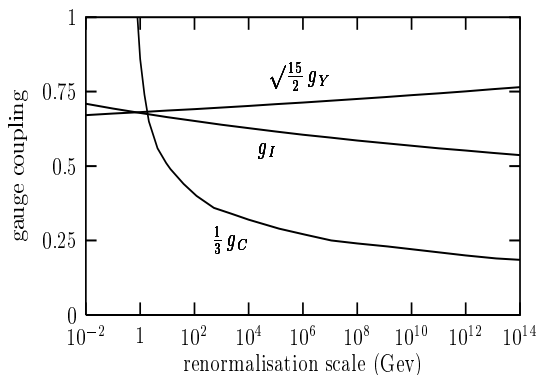


FIG. 1. Rescaled running gauge couplings.

B. The Dualised Standard Model

To construct a dualised standard model Chan and Tsou propose the elementary fermions are dyonically charged, with dynamics depending upon both an electric and magnetic gauge symmetry [5]

$$H_{SM} \times \widetilde{H}_{SM}. \quad (4)$$

Note an independent treatment of Abelian dualised gauge symmetry has been given by Kleinert [16].

The main implication of the dualised structure (4) is associated with the existence and properties of three standard model generations [6]. Following a theorem of 't Hooft, colour confinement implies dual colour $\widetilde{SU}(3)_C$ breaks to triviality [17]. Then a dual colour multiplet

$$\psi = (\psi^{\tilde{r}}, \psi^{\tilde{g}}, \psi^{\tilde{b}}),$$

splits into three components, with each component's mass determined by the details of the breaking. Interpreting this dual colour as a horizontal generational symmetry naturally leads to three generations of fermions from one generation of dual colour charged particles.

To describe symmetry breaking the relevant condensing scalar degrees of freedom must be identified. Chan and Tsou claim such scalar fields occur as frame vectors within the non-Abelian electric-magnetic duality. In this sense these scalar fields are interpreted as independent degrees of freedom arising naturally from the dualised nature of (4). Within a dualised standard model this gives two isospin doublets and three dual colour triplets.

This structure allows an estimation of fermion masses by constructing Yukawa couplings between the fermions and the dual colour scalar fields. Analogous to electroweak theory this is possible when only the isospin doublet fermions are dual colour charged. At tree level this coupling diagonalises into only one massive generation, which roughly approximates the standard model. To first order non-zero masses are induced for the other two generations [7]:

TABLE II. Particle mass predictions.

	calculation	experiment
m_c	1.327 GeV	1.0 – 1.6 GeV
m_s	173 MeV	100 – 300 MeV
m_μ	106 MeV	105.7 MeV
m_u	235 MeV	2 – 8 MeV
m_d	17 MeV	5 – 15 MeV
m_e	7 MeV	0.511 MeV

Note the poor match for the lightest generation, which they attribute to their approximation techniques. The CKM mixing angles also derive from the same inputs

$$|V_{rs}| = \begin{pmatrix} 0.9752 & 0.2215 & 0.0048 \\ 0.2210 & 0.9744 & 0.0401 \\ 0.0136 & 0.0381 & 0.9992 \end{pmatrix};$$

again these compare favourably with experiment

$$\begin{pmatrix} 0.9745 - 0.976 & 0.217 - 0.224 & 0.0018 - 0.0045 \\ 0.217 - 0.224 & 0.9737 - 0.9753 & 0.036 - 0.042 \\ 0.004 - 0.013 & 0.035 - 0.042 & 0.9991 - 0.9994 \end{pmatrix}.$$

C. Complementarity of the Dual and Dualised Standard Models

In this section we discuss how the above two models complement each other. That is, how their physical implications have no overlap, whilst their total implications explain most of the standard model. It is important to stress that these models are completely independent, and that they discuss different aspects of non-Abelian duality.

Firstly, the principle success of the dual standard model is to predict the electric charges for just one standard model generation, whilst it gives no explanation for three generations. Complementary to this the dualised standard model takes these electric charges as input, whilst deriving three generations.

Secondly, the dual standard model explains the origin of spin through considering dyons instead of monopoles. Complementary to this the dualised standard model takes these spins as input, whilst assuming the fermions are dyonic to derive three generations. Later we will see that the specific representations required to achieve these effects can be consistent.

Thirdly, no particle masses have been derived in the dual standard model, whilst this is a central aspect of the dualised standard model. Currently the only indication for the dual standard model mass scale is through the gauge unification in fig. 1, which suggests a few GeV.

Finally for electroweak symmetry breaking and confinement the dual standard model assumes the necessary scalar field structure. Complementary to this the dualised standard model derives such fields from the properties of non-Abelian duality.

We hope this gives some motivation for treating these two theories together. For further corroboration we now make some additional comments.

The point of both the dualised standard model and the dual standard model is to express the standard model in a simpler form. The dualised standard model does this by reducing the situation to essentially one generation of fermions. In the dual standard model one generation of fermions is understood to originate from gauge unification. In this sense the dual standard model reduces the fermions to simply a consequence of gauge interaction.

Finally the dual and the dualised standard model complement each other on a theoretical level. The dual standard model is based on the notion that electric particles can also be described as monopoles in the dual gauge potential. The dualised standard model is based on a quite different aspect of duality, where both electric and magnetic interactions are considered together as a dualised theory.

D. Combining the Dual and Dualised Standard Models

In the previous section we discussed how the dual and dualised standard models are complementary theories. This motivates that perhaps they should be combined together to give a full description of the standard model. A natural way to do this would be to dualise the dual standard model by somehow inducing colour charges on the monopoles.

However as the two models presently stand there are difficulties with this dualisation. This is because the con-

struction of dual colour Yukawa couplings requires dual colour matter assignments $\mathbf{3}_L, \mathbf{1}_R \times 3$, with only $(u, d)_L$ and $(\nu, e)_L$ dyonic [6]. That is, the dualised standard model derives three generations by postulating a dual colour structure analogous to electroweak isospin. The evidence for this are the rather accurate estimations of fermion masses and mixing angles [7].

However this structure is not compatible with dualising the dual standard model. There only one generation of u_R, d_R and e_R are derived; not the three required to construct dual colour Yukawa couplings. Instead, from a dual standard model perspective, it appears natural that all SU(5) monopoles should somehow gain dual colour charge; then three generations would originate solely from dual colour. Certainly the physical mechanism of dual colour breaking still appears to apply, although the fermion masses and mixing angles would not.

Perhaps an investigation of the effective couplings between monopoles may yield similar couplings; for instance if $\mathbf{3}_R$ is first broken to $\mathbf{1}_R \times 3$ then such Yukawa couplings can be constructed. In the dualised standard model these Yukawa couplings are effective anyway; since they are not gauge invariant unless derived from a non-renormalisable interaction [6].

Perhaps many of these issues relate to quantising the dual standard model (indeed we will see later there are other problems with quantisation). As a preliminary investigation one might determine whether such dual colour charges may naturally occur within the classical monopole theory. That is the subject of the next section.

III. DUALISING THE DUAL STANDARD MODEL

In the previous section we motivated that perhaps one should combine the dual standard model and the dualised standard model together. Within this section we give an illustration of the sort of methods that could be used.

Essentially we examine here only whether the most basic proposal in the dualised standard model is consistent with a dual standard model. That is whether the dual standard model can be naturally dualised such that every elementary particle has the dual colour required for three generations. In addition we check whether this procedure is consistent with the parity and angular momentum assignments of the elementary particles.

We should make it clear that there are some difficult issues when extrapolating the following to the fully quantised regime. These issues and some potential resolutions are discussed in sec. (IIID). For this reason the following represents an investigation of whether a consistent classical/semi-classical theory can be obtained.

A. SU(5) Monopoles in a Theta Vacuum

To start we consider the effects of a theta vacuum on the SU(5) monopole spectrum of sec. (II A). Such a theta vacuum has been motivated to play an important role in formulating a dual standard model [1, 2]. This is because the SU(5) monopole spectrum is parity invariant (unlike the standard model) unless a theta vacuum is included.

The effect of a theta term in the SU(5) gauge theory

$$\mathcal{L}_\theta = \frac{\theta g^2}{8\pi^2} \text{tr } \mathbf{E} \cdot \mathbf{B} \quad (5)$$

is to induce theta dependent electric charge [18] on the monopoles in table I. A simple way to see this is to consider the interaction of a monopole with a gauge field (ϕ, \mathbf{a}) . Following an argument of Coleman's [19] the electric and magnetic fields

$$\mathbf{E} = \nabla \phi, \quad \mathbf{B} = \nabla \wedge \mathbf{a} + \frac{1}{2g} \frac{\hat{\mathbf{r}}}{r^2} M, \quad (6)$$

are substituted into (5) to give, upon integration by parts,

$$L_\theta = \int d^3\mathbf{r} \mathcal{L}_\theta = -\frac{\theta g}{2\pi} \int d^3\mathbf{r} \delta^3(\mathbf{r}) \text{tr } \phi M. \quad (7)$$

But this is precisely the interaction between the gauge potential and an electric charge $Q_\theta = -\frac{\theta}{2\pi} M$. Consequently each monopole in table I gains a theta dependent electric charge, becoming a dyon

$$\mathbf{E} \sim -\frac{\theta g}{2\pi} \frac{\hat{\mathbf{r}}}{4\pi r^2} M, \quad \mathbf{B} \sim \frac{1}{2g} \frac{\hat{\mathbf{r}}}{r^2} M. \quad (8)$$

Here we are particularly interested in the effects of this theta vacuum on the interactions of these monopoles with electric charges. To be specific we consider the bosons associated with the components H_i , $i = 1, \dots, 5$, of a $\mathbf{5}$ scalar field. These source a non-Abelian electric field

$$\mathbf{E} = g \frac{\hat{\mathbf{r}}}{4\pi r^2} Q_i, \quad (9)$$

with Q_i the electric generator, which has the form

$$\begin{aligned} Q_1 &= \text{diag}\left(\frac{4}{5}, -\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}\right), \\ \dots &\quad \dots \\ Q_5 &= \text{diag}\left(-\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, \frac{4}{5}\right). \end{aligned} \quad (10)$$

Similarly the \bar{H} bosons have electric generators \bar{Q}_i of opposite sign. Then $\{Q_1, Q_2, Q_3\}$ form a colour triplet and $\{Q_4, Q_5\}$ form an isospin doublet; each with the appropriate colour and isospin charges.

Note that this scalar field H associated with the above bosons is directly relevant to constructing a dual standard model. In points (ii) and (iii) of sec. (II A) this field

is used to obtain fermions from bosons and for breaking dual colour.

Because of the global properties of charge around a monopole the theta induced charge is always associated with an Abelian interaction. That is, not all electric charges can be defined in the presence of a monopole (with some gaining an infinite energy string singularity), but the theta induced charge is necessarily well defined as a $U(1)_M$ interaction [20,21]. This induced charge then interacts with the H_i bosons through their $U(1)_M$ charge components.

The feature we wish to draw attention to is that the theta induced charge will attract some electric charges and repel others in a parity violating manner. Whether they are attracted or repelled depends upon whether the H_i bosons have positive/negative $U(1)_M$ charges. This provides a natural, parity violating mechanism for forming dyonic composites. Many of these have one-half angular momentum, as is necessary to construct a dual standard model.

This can be illustrated by considering the theta induced Coulomb potential between a monopole with generator M and a charge Q

$$V(r) \sim -\frac{\theta g^2 \text{tr } QM}{2\pi \cdot 4\pi r}, \quad r \gtrsim R_c. \quad (11)$$

Here both the charge and monopole are approximated as point sources outside the monopole core. Inside the core the magnetic field (and hence induced electric field) decreases continuously to zero by Gauss's law. Clearly this potential is binding/repulsive depending upon whether $\text{tr } QM$ is positive/negative.

That the potential (11) results in a parity violating spectrum of bound dyons is because of the even/odd properties of the electric/magnetic fields under parity inversion. Then $\mathcal{P} : (Q, M) \mapsto (Q, -M)$ takes a bound dyon into a non-bound state.

An interesting property of the stable dyons is that their angular momentum assignments also violate parity. For scalar electric charges the angular momentum of the resulting dyonic composites is

$$J_3 = \int d^3r [\mathbf{r} \wedge (\mathbf{E} \wedge \mathbf{B})]_3 = \frac{1}{2} \text{tr } QM, \quad (12)$$

with the monopole-charge axis orientated to \hat{x}_3 . Then their parity conjugates have the opposite angular momentum.

B. The Dual Standard Model in a Theta Vacuum

We now apply the above properties of a theta vacuum to the construction of a dual standard model. The central idea is to use the theta binding effect to naturally form a parity violating spectrum of $SU(5)$ dyons, all of which have one-half angular momenta.

As well as trying to achieve angular momentum assignments compatible with the standard model we would also like the resulting dyon spectrum to be compatible with obtaining three generations through the methods of Chan and Tsou. To help along these lines we take some indications from the dualised standard model. There they require dual colour to be broken, whilst dual isospin appears to be confining, with a large confinement scale (say over a hundred GeV) to not be presently observed.

Therefore we do not consider $SU(5)$ dyons with electric isospin, as in the dualised standard model these are confined into very heavy dual isospin hadrons. As we will see this conveniently simplifies the following calculations.

Such composite dyons can be formed by combining the monopoles in table I with the charges $\{Q_1, Q_2, Q_3\}$ in (10). However these are not the only dyons present in the dual standard model; there are also non-Abelian analogues of the Julia-Zee dyon [22]. In principle the theta induced charge can also bind their electric charge to the monopoles.

The description of these monopole gauge excitations is quite involved and we refer to ref. [23] for a fuller discussion. Care has to be taken with the global properties of electric charge, because not all charges are well defined around a magnetic monopole. The dyon spectrum is obtained upon performing a semi-classical quantisation of the global electric degrees of freedom around a monopole. This results in the following spectrum of possible electric charges, with colour, isospin and hypercharges defined through $Q = q_C T_C + q_I T_I + q_Y T_Y$:

TABLE III. Gauge excitations of the monopoles.

diag Q	q_C	q_I	q_Y	allowed on
(0, 0, 1, -1, 0)	1	$\frac{1}{2}$	$\frac{1}{3}$	all
(0, 1, 1, -1, -1)	-1	0	$\frac{2}{3}$	$\bar{d}, (\bar{\nu}, \bar{e}), u, \bar{e}$
(1, 1, 1, -2, -1)	0	$-\frac{1}{2}$	1	$(\bar{\nu}, \bar{e}), u, \bar{e}$
(1, 1, 2, -2, -2)	1	0	$\frac{4}{3}$	$(\bar{\nu}, \bar{e}), u, \bar{e}$
(1, 2, 2, -3, -2)	-1	$\frac{1}{2}$	$\frac{5}{3}$	$(\bar{\nu}, \bar{e}), u, \bar{e}$
(2, 2, 2, -3, -3)	0	0	2	$(\bar{\nu}, \bar{e}), u, \bar{e}$

Here we restrict our attention to the lower charged and therefore least energetic excitations. Of these there are three that are uncharged under isospin

$$Q_C = \text{diag}(0, 1, 1, -1, -1), \quad Q_{C'} = \text{diag}(1, 1, 2, -2, -2), \\ Q_Y = \text{diag}(2, 2, 2, -3, -3). \quad (13)$$

From now on we will take these charges as input, and not consider their origin from the semi-classical quantisation.

Now we need to determine the angular momenta of these gauge excited dyons. For this there are two situations:

(i) Spherically symmetric dyons have vanishing angular momentum. There is a simple criterion for determining

whether the dyons are spherically symmetric from their (Q, M) charge, as described in sec. (VIII) of ref. [23].

(ii) Otherwise dyons will have angular momentum, although there are many issues that have not been fully understood. As a simple model for calculating their angular momentum we consider Q to be composed of two components $Q = Q_0 + Q_s$, where Q_0 defines a spherically symmetric dyon. Then the angular momentum originates from Q_s , which we interpret as a single gauge boson in the background of the monopole. This gives

$$J_3 = \begin{cases} \frac{1}{2} \text{tr } Q_s M - 1, & \text{tr } Q_s M \geq 0, \\ \frac{1}{2} \text{tr } Q_s M + 1, & \text{tr } Q_s M \leq 0, \end{cases} \quad (14)$$

in which we have included the spin of the gauge boson as being energetically orientated opposite to the magnetic field [23] and taken the monopole-charge axis as \mathbf{x}_3 .

It should be noted that there are many issues with gauge excitations that have not been fully understood. However the above configurations are present in the SU(5) monopole theory, and one may use Goldhaber's argument [14] to show they are fermionic. It is possible that (14) is only valid for some gauge excitations, although we have checked that those dyons in table IV below are compatible with the semi-classical analysis of Dixon [24].

From this we can determine the angular momenta of the appropriate dyons in the dual standard model:

TABLE IV. Angular momenta of the dyons.

n	multiplet	Q_1	Q_2	Q_3	Q_C	Q'_C	Q_Y
1	(u, d)	0	0	$\frac{1}{2}$	—	—	—
2	\bar{d}	0	$\frac{1}{2}$	$\frac{1}{2}$	0	—	—
3	$(\bar{\nu}, \bar{e})$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	± 1	0
4	u	$\frac{1}{2}$	$\frac{1}{2}$	1	$-\frac{1}{2}$	0	0
6	\bar{e}	1	1	1	± 1	± 1	0
6	\bar{e}^*	$\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{1}{2}$	0	0

In this table we also include an \bar{e}^* monopole with $M = (1, 1, 4, -3, -3)$, which has extra non-topological magnetic charge. Vachaspati and Steer have motivated that this non-topological degree of freedom relates to the internal structure of the monopole, so that the long range magnetic interactions are unaffected. For more details we refer to their paper [8].

For the scalar boson-monopole composites only the states (Q, M) with non-zero angular momentum can be bound configurations. This is because both the binding potential (11) and angular momentum (12) are proportional to $\text{tr } QM$. The charge conjugates $(-Q, -M)$ have also the same stability and angular momenta, whilst the parity conjugates $(Q, -M)$ have the opposite stabilities.

In conclusion this mechanism has been of some success within the dual standard model. Certainly all of the monopoles become naturally dyonic, with a spectrum

of non-zero angular momenta that violates parity maximally. Also all are colour charged, as is consistent with a dualised standard model. However there are problems: (i) There is no \bar{e} dyon with one-half angular momentum. To construct such a state requires two quanta of H , which is problematic as such a dyon possesses dual isospin.

(ii) As well as the desired standard model states there are other angular momentum analogues for the $(\bar{\nu}, \bar{e})$, u and \bar{e} dyons. Certainly no such states have been observed in the standard model.

A way of solving problem (i) is to instead consider the \bar{e}^* monopole, which has dyons with one-half angular momentum. Later we will see that the \bar{e} dyons have a complicated energy spectrum at non-zero theta. This raises the possibility that a dyon with non-topological magnetic charge could be the admissible state.

Problem (ii) is less straightforward. It seems that some energetic criterion should be applied. We examine this in the next section.

C. Dualising the Dual Standard Model

As the above mechanism stands there is another reason why all the above composites cannot represent standard model fermions. One should require each composite's mass to be less than their possible decay products, since only then are the dyons absolutely stable to decay. Note that similar ideas have been proposed in ref. [8], although the following discussion is very different from that.

In this section we will take the charges and spins of the dyons as input and consider only the classical charge-monopole interactions. This is because there are some difficult issues associated with a fully quantised treatment, as discussed in the next section.

The SU(5) monopole masses can be estimated in a fairly simple way from their solitonic properties. Their scalar core energy and magnetic mass are determined by the coupling g , vev v , and core-size R_c

$$\mathcal{E}_s \sim \frac{1}{2} \int_{r < R_c} d^3r \text{tr } |D\Phi|^2 \sim 2\pi R_c v^2, \quad (15)$$

$$\mathcal{E}_B \sim \frac{1}{2} \int_{r > R_c} d^3r \text{tr } B^2 \sim \frac{2\pi}{g^2 R_c} |M|^2, \quad (16)$$

where $|M|^2 = \text{tr } M^2$. For simplicity we approximate the scalar and gauge core sizes as equal, which does not appreciably effect the central result (20) below. An equilibrium between the scalar and magnetic energies leads to

$$R_c \sim \frac{|M|}{gv}, \quad m_{\text{mon}} \sim \mathcal{E}_B + \mathcal{E}_s \sim \frac{4\pi v}{g} |M|. \quad (17)$$

It is interesting that the electromagnetic mass essentially determines the monopole's mass m_{mon} . The problem of electromagnetic mass has a long history (see ref. [25]);

for instance it diverges in many situations. For the above solitons the role of electromagnetic mass is clear: it simply constitutes half of the monopole's mass.

The electromagnetic mass is also central to calculating the soliton's mass in a theta vacuum. Then the total mass is the sum of m_{mon} in (17) and the mass in the theta induced electric field (8)

$$\mathcal{E}_E \sim \frac{1}{2} \int_{r>R_c} d^3r \text{tr} E^2 \sim \frac{\theta^2 g^3 v}{32\pi^3} |M|. \quad (18)$$

An interesting point is that a dyon's electric charge can cancel off part of the theta induced electric field. This will decrease the electric mass of the dyon. There are many issues with this observation, but let us explore the consequences for the stable dyon spectrum.

Considering a dyon (Q, M) in a theta vacuum,

$$\mathbf{E} \sim g \frac{\hat{\mathbf{r}}}{4\pi r^2} (Q - \frac{\theta}{2\pi} M), \quad \mathbf{B} \sim \frac{1}{2g} \frac{\hat{\mathbf{r}}}{r^2} M. \quad (19)$$

Then the electric mass of this dyon is

$$\mathcal{E}_E \sim \frac{1}{2} \int_{r>R_c} d^3r \text{tr} E^2 \sim \frac{g^3 v}{8\pi |M|} \text{tr} (Q - \frac{\theta}{2\pi} M)^2, \quad (20)$$

whilst the magnetic mass stays the same.

For the dyons in table IV we now plot all of their electric energies with theta in figs. 2 to 7. Those dyons not included on the figures, which includes the different gauge orientations, have been verified to not be of least energy.

In conclusion for $\theta \in (\frac{4}{5}\pi, \frac{10}{11}\pi)$ the states with least electric mass are:

TABLE V. States of least electric mass for $\theta \in (\frac{4}{5}\pi, \frac{10}{11}\pi)$.

n	diag Q	diag M	J_3	fermion
1	$(-\frac{1}{5}, -\frac{1}{5}, \frac{4}{5}, -\frac{1}{5}, -\frac{1}{5})$	$(0, 0, 1, -1, 0)$	$\frac{1}{2}$	$(u, d)_L$
2	$(-\frac{1}{5}, -\frac{1}{5}, \frac{4}{5}, -\frac{1}{5}, -\frac{1}{5})$	$(0, 1, 1, -1, -1)$	$\frac{1}{2}$	\bar{d}_L
3	$(0, 1, 1, -1, -1)$	$(1, 1, 1, -1, -2)$	$-\frac{1}{2}$	$(\bar{\nu}, \bar{e})_R$
4	$(0, 1, 1, -1, -1)$	$(1, 1, 2, -2, -2)$	$-\frac{1}{2}$	u_R
6	$(0, 1, 1, -1, -1)$	$(2, 2, 2, -3, -3)$	1	—
6	$(0, 1, 1, -1, -1)$	$(1, 1, 4, -2, -2)$	$\frac{1}{2}$	\bar{e}_L^*

Thus if the criterion for selecting relevant states is the dyon's electric mass then this does give the required spectrum, with all states having $|J_3| = \frac{1}{2}$. We stress that in no way was this result necessary or predetermined; the dynamics just happened to give the desired answer. There are difficulties with the \bar{e} states, but \bar{e}^* may be less massive and these two monopoles differ only by their internal structure [8].

Note that all dyons in table V are electrically colour charged, and transform fundamentally under electric colour. Therefore their dyonic charges are compatible with Chan and Tsou's interpretation of three generations.

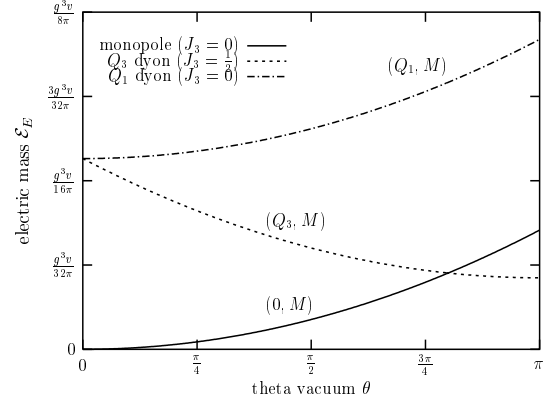


FIG. 2. Electric mass of the (u, d) monopole and dyons (Q_3, M) and (Q_1, M) . For $\theta \in (\frac{4}{5}\pi, \pi)$ the (Q_3, M) dyon with $J_3 = \frac{1}{2}$ has least electric mass.

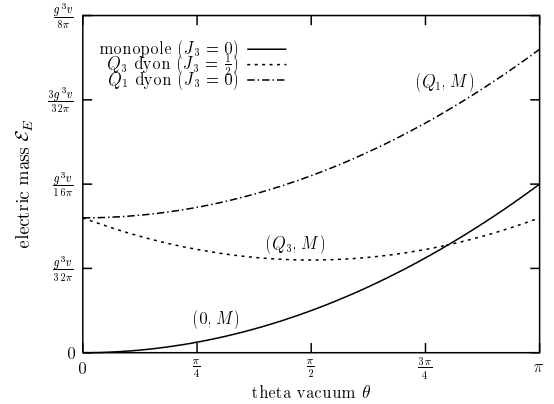


FIG. 3. Electric mass of the \bar{d} monopole and dyons (Q_3, M) and (Q_1, M) . For $\theta \in (\frac{4}{5}\pi, \pi)$ the (Q_3, M) dyon with $J_3 = \frac{1}{2}$ has least electric mass.

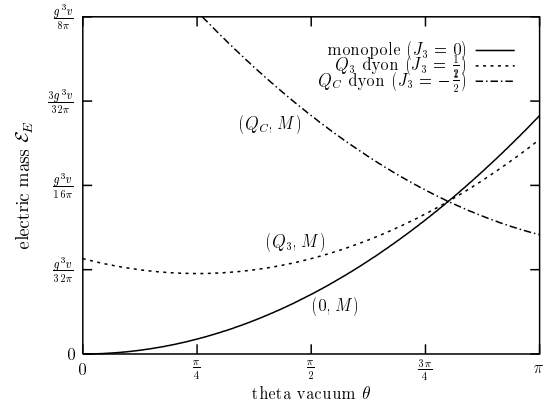


FIG. 4. Electric mass of the $(\bar{\nu}, \bar{e})$ monopole and dyons (Q_3, M) and (Q_C, M) . For $\theta \in (\frac{4}{5}\pi, \pi)$ the (Q_C, M) dyon with $J_3 = -\frac{1}{2}$ has least electric mass.

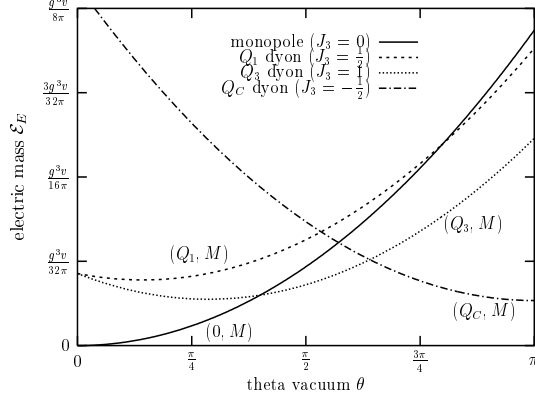


FIG. 5. Electric mass of the u monopole and dyons (Q_1, M) , (Q_3, M) and (Q_C, M) . For a range of theta the (Q_C, M) dyon with $J_3 = -\frac{1}{2}$ has least electric mass.

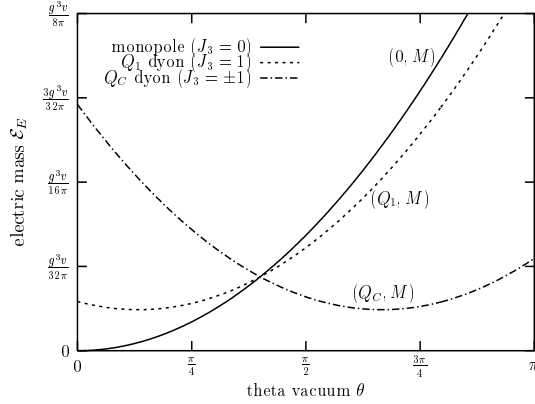


FIG. 6. Electric mass of the \bar{e} monopole and the dyons (Q_1, M) and (Q_C, M) . For a range of theta the (Q_C, M) dyon with $J_3 = 1$ has least electric mass.

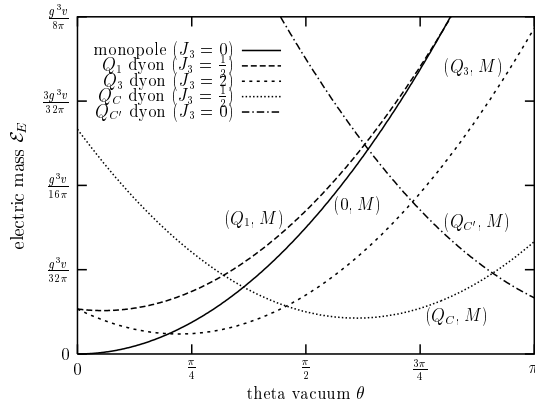


FIG. 7. Electric mass of the \bar{e}^* monopole and the dyons (Q_1, M) , (Q_3, M) , $(Q_{C'}, M)$ and (Q_C, M) . For $\theta \in (\frac{4}{5}\pi, \frac{10}{11}\pi)$ the (Q_C, M) dyon with $J_3 = \frac{1}{2}$ has least electric mass.

An interesting and unexpected bonus of the above calculation is that we also appear to have obtained an association between the fermion's chirality assignments and the sign of the dyon's angular momenta. In the high momentum limit $p \gg m$ the angular momentum of a chiral fermion is unambiguously determined through the helicity projection operator $\Pi^\pm(\mathbf{p}) = \frac{1}{2}(1 \pm \gamma_5)$. In that limit its angular momentum along the direction of motion is

$$J_3 \psi_L = \frac{1}{2} \psi_L, \quad J_3 \psi_R = -\frac{1}{2} \psi_R. \quad (21)$$

Therefore table V also gives a correspondence between these angular momenta and those of the associated dyons. Again the pattern $\{+\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}\}$ occurred through the specific dynamics of the situation.

Thus the question is: does this least electric mass criterion justify that the dyon is stable? Fortunately there is at least one situation where this appears to be so.

At strong electric coupling $g^2/4\pi \gg 1$ in a theta vacuum a monopole's electric mass is the dominant mass contribution. Then the magnetic mass, the gauge boson masses, and the scalar boson masses may be consistently taken to be much smaller than this electric mass. If so then figs. 2 to 7 appear to represent the dyon's masses, of which table V contains those with least mass.

However there are then some difficult issues, mainly relating to quantisation. These are discussed in the next section.

D. Issues and Interpretation

In the previous section we saw that a reasonable spectrum of dyons could be derived providing the gauge theory is strongly coupled. However there are then some difficult issues, as we now explain.

When the gauge theory is strongly coupled the nature of the monopoles is different from at weak coupling. This can be seen by comparing their core size R_c with their Compton wavelength $\lambda \sim m^{-1}$

$$\lambda/R_c \sim g^2/4\pi. \quad (22)$$

Thus at weak coupling the monopoles are classical soliton configurations, whilst at strong coupling they are fully quantum mechanical.

At first sight this seems promising for the dual standard model because the observed fermions are fully quantum mechanical. That is, the observed elementary particles are not quantised semi-classically but are fully quantised through methods such as the path-integral formalism. Then it would seem as if a proper quantisation of the dual standard model should yield a quantum field theory similar to the standard model.

However some difficult issues arise when extending the calculations in secs. (III A-III C) to the fully quantum regime at strong coupling. Specifically:

(i) Whilst the theta induced charge is valid at strong coupling in a classical context, the effects of quantisation are not known.

(ii) Also, a substantial theta vacuum would give a strong \mathcal{CP} problem. The strong interactions are time reversal symmetric to large accuracy and a large θ would appear to be at odds with this. Additionally, although we have not discussed weak \mathcal{CP} violation in this work, the induced violation would be too large there as well.

It is beyond the scope of this paper to fully address these issues. Indeed, as we discuss below, their resolution may require a detailed understanding of the quantisation of solitons at strong coupling; a subject that is poorly understood. We discuss now a couple of different perspectives on these issues.

Firstly it could transpire that the dual standard model is only appropriate as a classical, effective description of the elementary particles. This would be analogous to the Skyrme model, in which baryons are accurately described as solitons of a classical field theory [26].

It is worth commenting that the modern interpretation of the Skyrme model is as a consequence of large N QCD [27], whereas the dual standard model is motivated by $SU(5)$ unification and duality. Otherwise the Skyrme model and dual standard model share many similar features and outlook; indeed, many of Skyrme's original motivations also apply to the dual standard model [28].

A second perspective is to tackle the above issues through a full quantisation of the dual standard model. Unfortunately, the techniques to carry out such a program have not been developed, so that no definitive conclusions can presently be made. However some tentative suggestions for resolving (i) and (ii) might then be:

(i) Perhaps the stable dyons can be represented by a second quantised fermionic field theory. In this sense the field theory may be largely insensitive to the internal details of the dyon, somewhat in analogy to how proton-electromagnetic interactions are described by QED, even though the proton has a quark substructure.

(ii) If a full quantisation of the $SU(5)$ solitons was to yield a reasonable description of the standard model fermions then an $O(1)$ contribution to the theta angle should arise from the determinant of the resulting quark mass matrix. Perhaps this could be arranged to cancel with the classical part used in this paper. In some sense this situation arises naturally in the standard model, since a suitable $\theta F\tilde{F}$ term is required to cancel this contribution; however here we interpret this necessary theta term as a starting ingredient of the dual standard model.

Whatever the interpretation we wish to stress that the classical arguments given in this paper do appear to produce the desired angular momentum assignments of the elementary particles. This is on top of the other successful features of the dual standard model.

IV. CONCLUSION

In this paper we have stressed that the dual and dualised standard models are complementary theories. That is, their implications have no overlap, whilst taking their consequences together appears to yield an explanation for most of the standard model. For instance the dual standard model explains the properties of one generation of standard model fermions as solitons originating from $SU(5)$ gauge unification; whilst the dualised standard model explains the properties of three generations as originating from a dualised fermion spectrum.

A central aspect of this paper is the suggestion that inclusion of a theta vacuum naturally combines these two theories together. Such a theta vacuum is expected to be important for a dual standard model because it should play a role in introducing parity violation. We have shown that in addition to this it has the effect of dualising the soliton spectrum. In doing so it also suggests an explanation for the chirality assignments of the elementary particles; a feature that cannot be derived in either of the original dual or dualised standard models.

There are difficult issues associated with the interpretation of these calculations at strong coupling. Whilst the arguments are well motivated classically, it is beyond the scope of this paper to carry them over to the fully quantised regime. We note, however, that the methodology is natural and the conclusions are consistent with the standard model.

If a consistent quantisation of the dual standard model does allow it to be naturally dualised by a theta vacuum, then perhaps a very simple theory of unification may ensue. In that instance particle and gauge unification could consist of merely a broken dual theta-gauge theory $\tilde{S}U(5) \rightarrow \tilde{S}U(3)_C \times \tilde{S}U(2)_I \times \tilde{U}(1)_Y/\mathbb{Z}_6$, with *all* presently observed particle properties occurring within the resulting soliton spectrum.

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- [1] T. Vachaspati, Phys. Rev. Lett. **76** (1996) 188 [hep-ph/9509271].
 - [2] H. Liu and T. Vachaspati, Phys. Rev. **D56** (1997) 1300 [hep-th/9604138].

- [3] T. Vachaspati, Phys. Lett. **B427** (1998) 323 [hep-th/9709149].
- [4] A. S. Goldhaber, Phys. Rept. **315** (1999) 83 [hep-th/9905208].
- [5] H. Chan and S. T. Tsou, Int. J. Mod. Phys. **A14** (1999) 2139 [hep-th/9904102]; see also S. T. Tsou, hep-th/0006178.
- [6] H. Chan and S. T. Tsou, Phys. Rev. **D57** (1998) 2507 [hep-th/9701120]; H. Chan and S. T. Tsou, Acta Phys. Polon. **B28** (1997) 3041 [hep-ph/9712436]; H. Chan, J. Bordes and S. Tsou, Int. J. Mod. Phys. **A14** (1999) 2173 [hep-ph/9809272].
- [7] J. Bordes, H. Chan, J. Faridani, J. Pfaudler and S. Tsou, Phys. Rev. **D58** (1998) 013004 [hep-ph/9712276].
- [8] T. Vachaspati and D. A. Steer, hep-th/0005243.
- [9] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438.
- [10] H. Chan and S. T. Tsou, Nucl. Phys. **B189** (1981) 364.
- [11] P. W. Higgs, Phys. Rev. **145** (1966) 1156; T. W. B. Kibble, Phys. Rev. **155** (1967) 1554.
- [12] C. Gardner and J. Harvey, Phys. Rev. Lett. **52** (1984) 879.
- [13] R. Jackiw and C. Rebbi, Phys. Rev. Lett. **36** (1976) 1116; P. Hasenfratz and G. 't Hooft, Phys. Rev. Lett. **36** (1976) 1119.
- [14] A. S. Goldhaber, Phys. Rev. Lett. **36** (1976) 1122.
- [15] N. F. Lepora, JHEP **0002** (2000) 036 [hep-ph/9910493].
- [16] H. Kleinert, Int. J. Mod. Phys. **A7** (1992), 4693; H. Kleinert, Phys. Lett. **B 246** (1990), 127; H. Kleinert, Phys. Lett. **B 293** (1992), 168.
- [17] H. Chan and S. T. Tsou, Phys. Rev. **D56** (1997) 3646 [hep-th/9702117]; G. 't Hooft, Nucl. Phys. **B190** (1981) 455.
- [18] E. Witten, Phys. Lett. **B86** (1979) 283.
- [19] S. Coleman, HUTP-82/A032 *Lectures given at Int. Sch. of Subnuclear Phys., Erice, Italy, 1981*.
- [20] P. Nelson, Phys. Rev. Lett. **50** (1983) 939; P. Nelson and A. Manohar, Phys. Rev. Lett. **50** (1983) 943; A. Abouelsaood, Nucl. Phys. **B226** (1983) 309; P. Nelson and S. Coleman, Nucl. Phys. **B237** (1984) 1.
- [21] A. P. Balachandran, G. Marmo, N. Mukunda, J. S. Nilsson, E. C. Sudarshan and F. Zaccaria, Phys. Rev. **D29** (1984) 2919; A. P. Balachandran, G. Marmo, N. Mukunda, J. S. Nilsson, E. C. Sudarshan and F. Zaccaria, Phys. Rev. **D29** (1984) 2936.
- [22] B. Julia and A. Zee, Phys. Rev. **D11** (1975) 2227.
- [23] N. F. Lepora, hep-ph/0008322.
- [24] L. J. Dixon, Nucl. Phys. **B248** (1984) 90.
- [25] See, for instance, chapter 28 of: R. Feynman, R. Leighton and M. Sands, Volume II, Addison-Wesley Publishing (1964).
- [26] T. H. R. Skyrme, Proc. Roy. Soc. (London) **A260** (1961) 127; Nucl. Phys. **B3** (1962) 556; J. Math. Phys. **12** (1971) 1735.
- [27] G. 't Hooft, Nucl. Phys. **B72** (1974) 461; **B75** (1975) 461; E. Witten, Nucl. Phys. **B160** (1979) 57.
- [28] See, for instance: T. H. R. Skyrme, Int. J. Mod. Phys. **A3** (1988) 2745. Talk reconstructed by I. Aitchison.